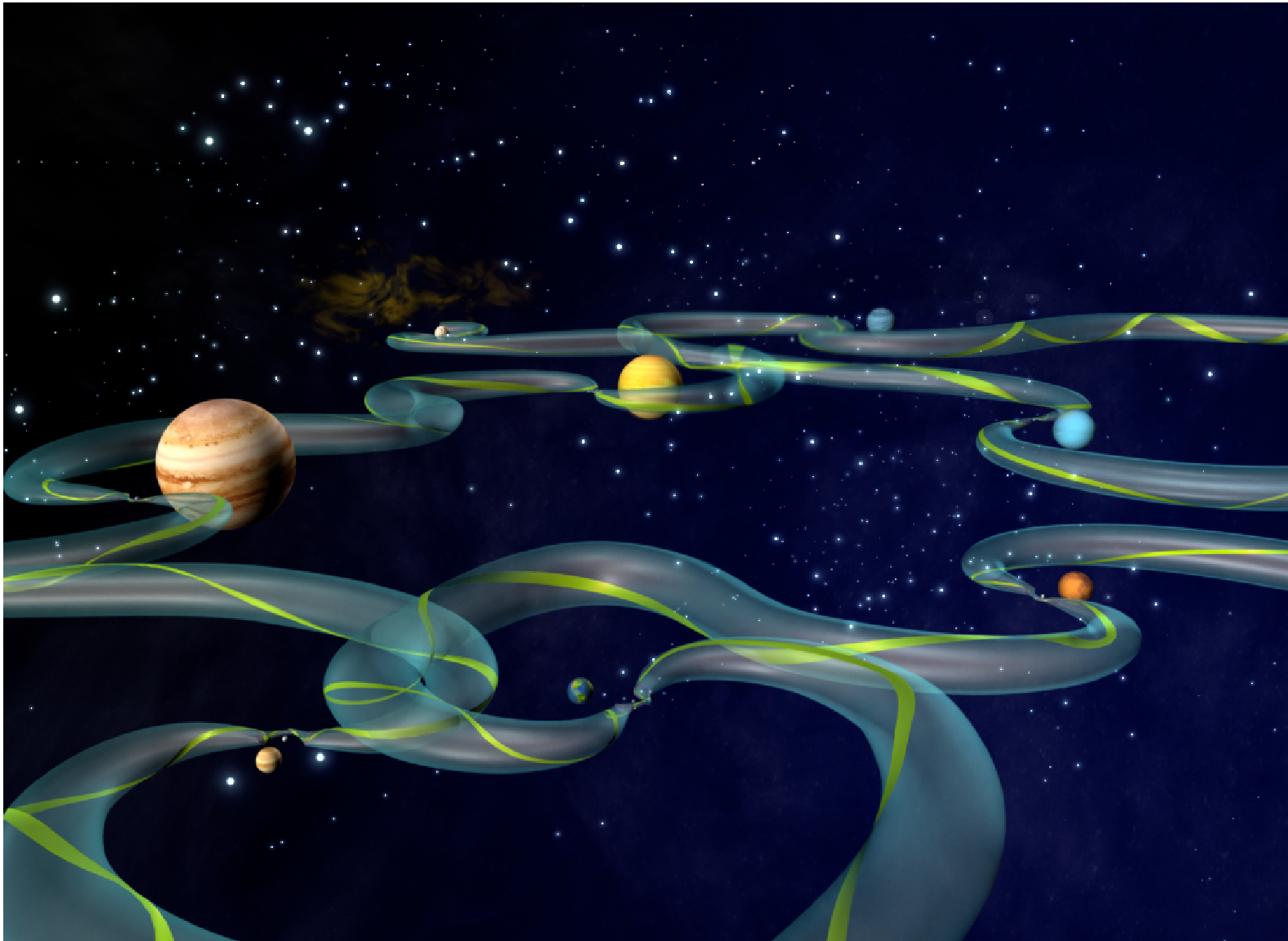


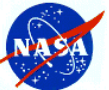
EPISODE: Software for Trajectory Generation and Mission Design



Science Enabling Technologies in Trajectory Design



- “History” of trajectory design - patched 2 body problems (i.e. Earth orbit, Lunar transfer, Voyager, etc.) We are now entering a new era of mission design utilizing 2 recent advances: 1) IPS, and 2) low thrust.
- Invariant manifolds provide IPS - can jump on and off manifolds which intersect in configuration space with delta V (jump in momentum). In fact there are configuration space intersecting manifolds whose energy difference is not that large (i.e. small delta V required to make a jump), so provides efficient transport throughout phase space.
- Why is the IPS useful? These invariant manifolds, when strung together, can provide cheap, almost free transport, through a large configuration space region (reachable set of free trajectories is very large!)
- IPS has allowed missions such as Genesis, WMAP, Planck, etc. to reach ideal environments for their science targets with affordable cost
- Another technology - low-thrust...
- Low-thrust is potentially mission enabling, allowing greater science return from the same mission (example - Jupiter tour, JIMO, in which a single spacecraft visits Callisto, Ganymede, and Europa in succession, would require $DV = 25 \text{ km/s}$! Ref. Whiffen and Lam, AAS 06-186)
- Now time to perfect the art of optimal open and closed loop control utilizing these mission enabling technologies



Problem Context

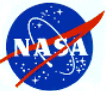


Project Objective:

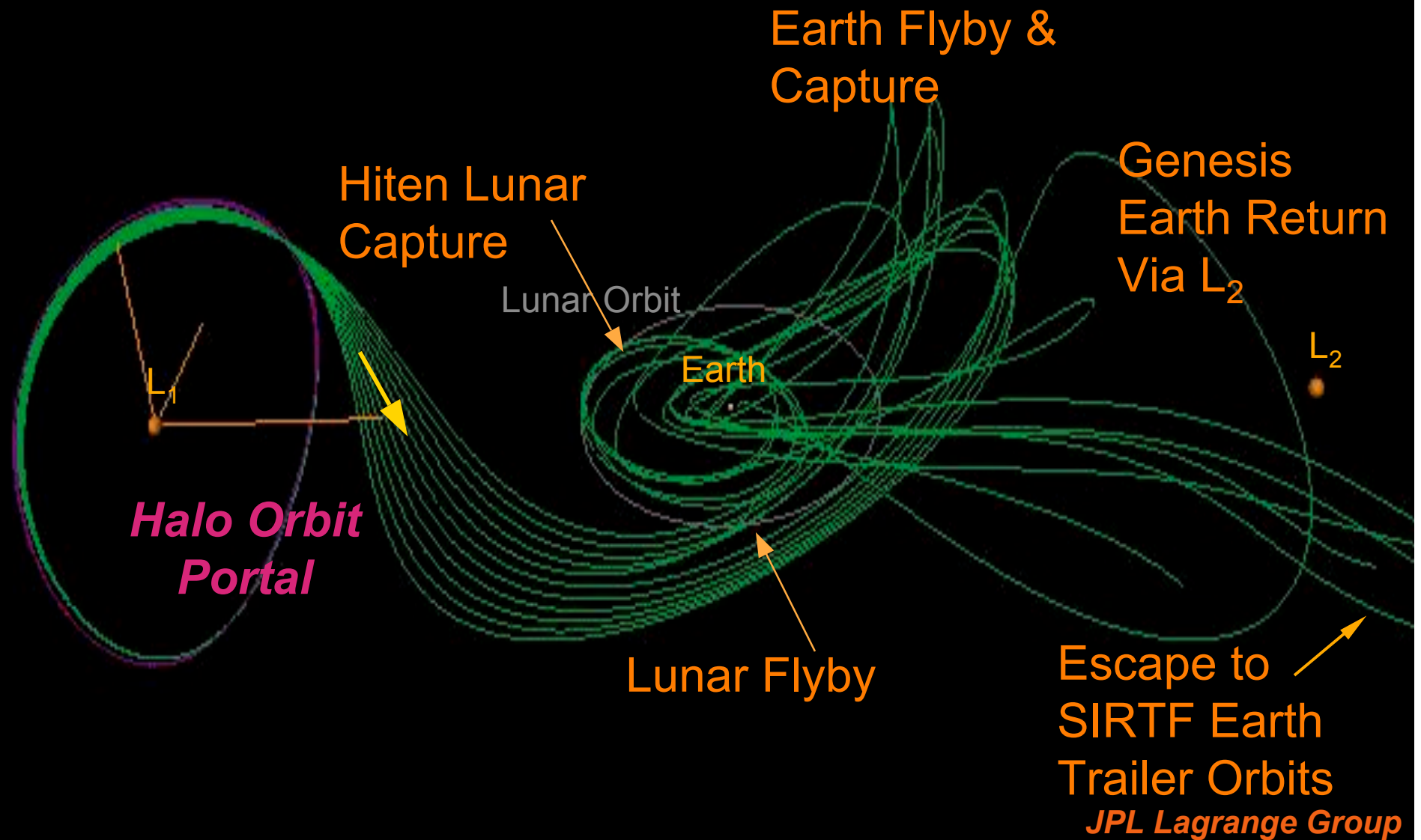
“We will develop the software named **EPISODE** (Evaluation of the Posterior for the Inference of Solutions of Ordinary Differential Equations) for an implementation of a probabilistic (Bayesian) approach for reasoning about dynamical systems in the presence of uncertainty with application to intelligent mission simulation, trajectory generation, and nonlinear continuous trajectory control, *in order to increase life cycle effectiveness and efficiency of the Science Mission Directorate research endeavors, in particular: 1) to reduce mission development time, risk, and cost through advanced simulation and design capabilities, and 2) to increase mission duration and reliability through autonomous operations and control.*”

NASA Relevance:

“We propose to approach the problems of trajectory generation and nonlinear continuous trajectory control in the presence of uncertainty as problems of statistical inference, and we will develop the software EPISODE needed to implement this approach. The proposed work *directly addresses several of NASA’s strategic objectives as outlined in Table 1 of the Summary of Solicitation of this NRA*, including *1) Undertake robotic and human lunar exploration to further science and to develop and test new approaches, technologies, and systems to enable and support sustained human and robotic exploration of Mars and more distant destinations, 2) Conduct robotic exploration across the Solar System for scientific purposes and to support human exploration – in particular, explore Jupiter’s moons, asteroids, and other bodies to search for evidence of life, to understand the history of the Solar System, and to search for resources, 3) Develop and demonstrate power generation, propulsion, life support, and other key capabilities required to support more distant, more capable, and/or longer duration human and robotic exploration of Mars and other destinations.*”



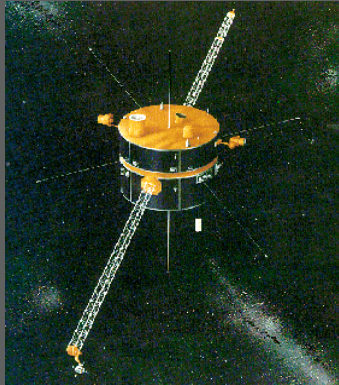
Validation of the Inter-Planetary Superhighway Trajectory Design Concept



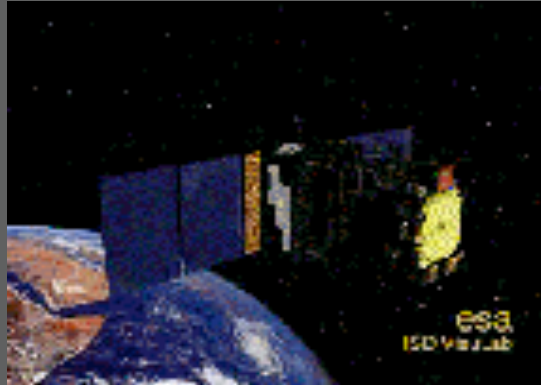
Current Libration Missions



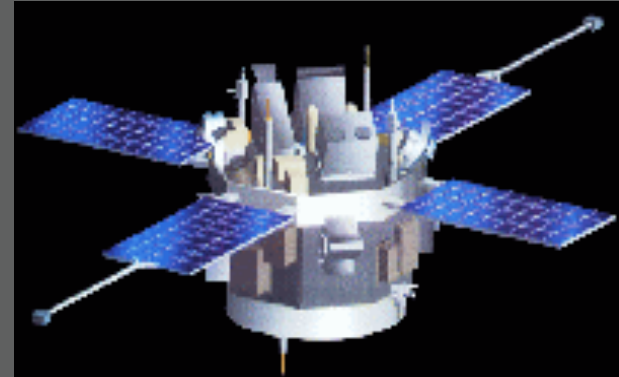
Goddard Space Flight Center



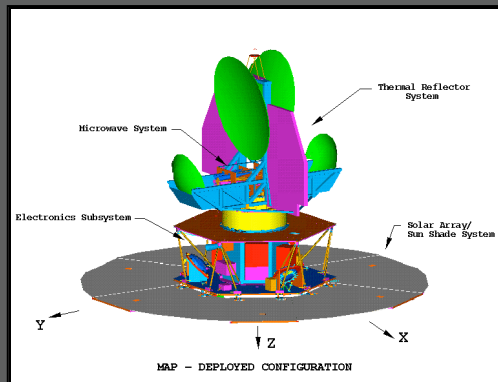
WIND



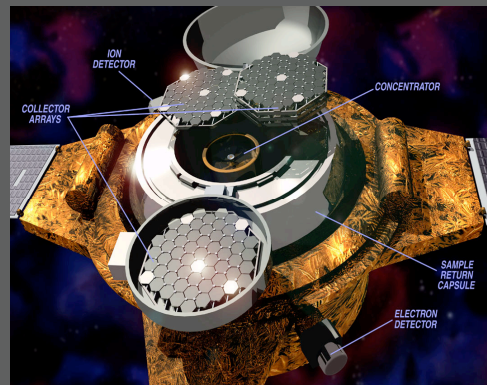
SOHO



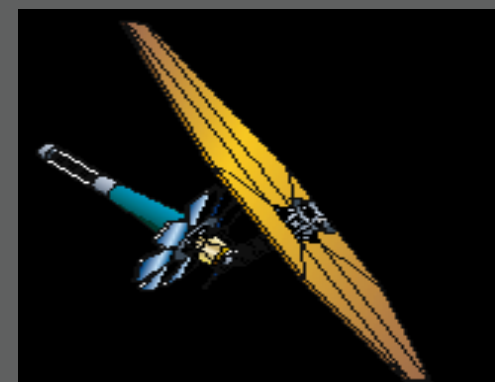
ACE



MAP



GENESIS

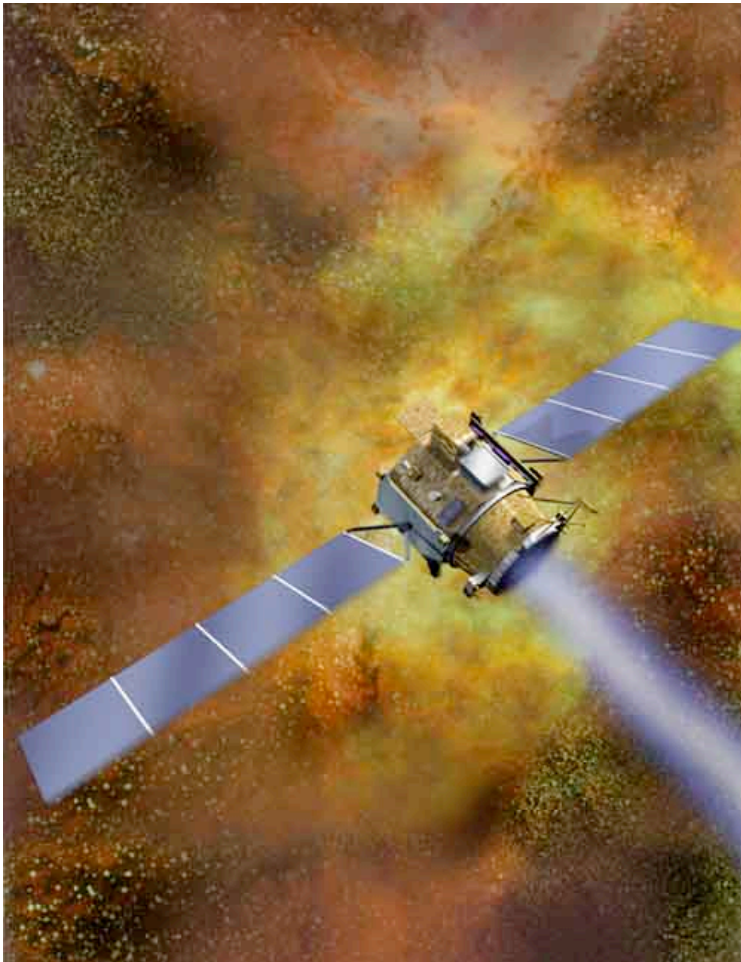


NGST

Courtesy of D. Folta, GSFC



Low-Thrust Propulsion



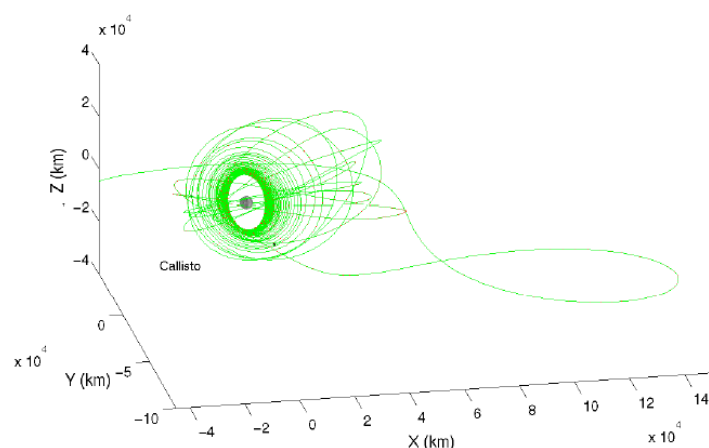
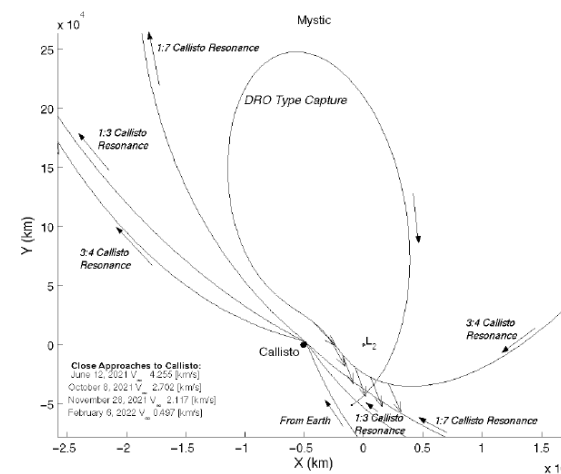
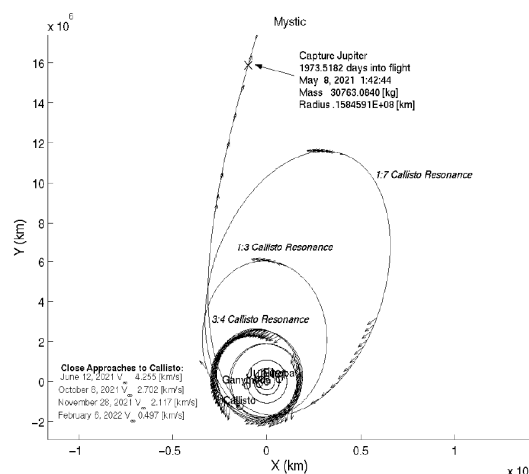
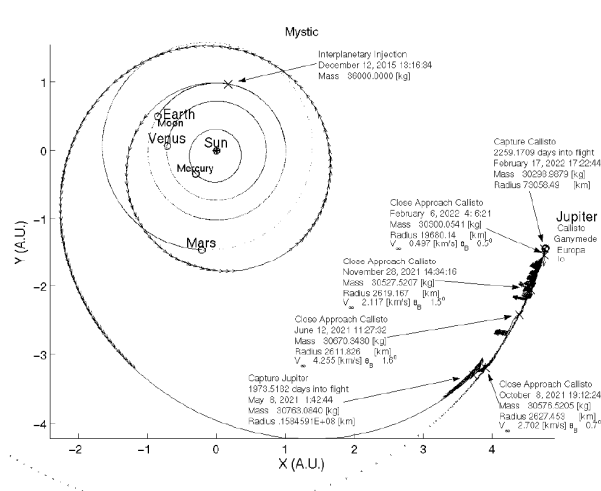
<http://nmp.nasa.gov/ds1/>

- Deep Space 1 provided successful flight validation of electric ion propulsion (low-thrust)
- Enabling for mission concepts such as JIMO, which is motivated by scientific interest in Jovian system, but requires DV of 25 km/sec to visit the moons of Callisto, Ganymede, and Europa.
- While low-thrust provides larger possible DV, corresponding trajectory design much more difficult due to potential instability and low control authority
- Example: JIMO reference trajectory VERY complex, and a very impressive computational feat accomplished by Whiffen and Lam using their high-fidelity design tool “Mystic”.

JIMO, Phase 1 of 3: Earth to Callisto



“The Jupiter Icy Moons Orbiter Reference Trajectory”, Gregory J. Whiffen, Try Lam, AAS 06-186



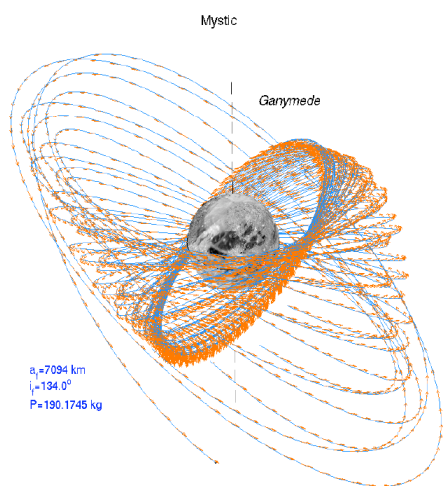
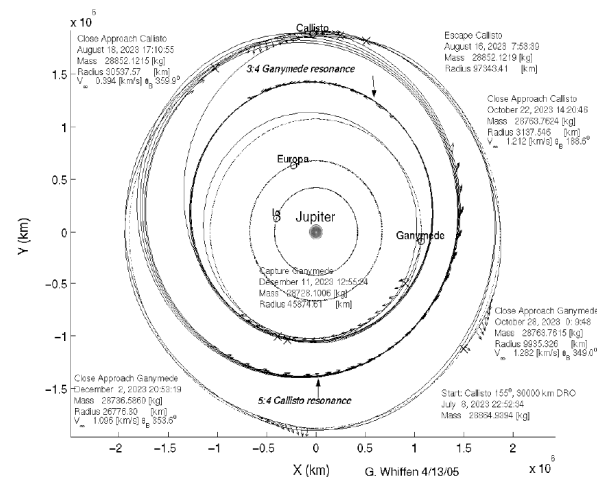
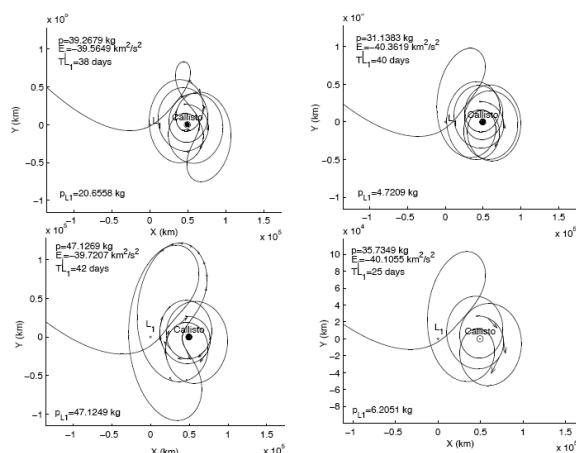
- Earth to Callisto involves Earth to Jupiter capture (approx. 6 years), followed by sequence of resonances to DRO capture, and low-thrust spiral down to Callisto
- Initial low-fidelity trade study for the Earth to Jupiter capture using MALTO (patched conic approximation) and VARITOP (two body).
- **Thousands of trajectories computed** to explore gravity assists from Venus, Earth, and Mars as well as study departure and arrival times).
- Minimum stay in Callisto science orbit 120 days - challenging to find stable enough orbit, as loss of ion engine could result in impact in hours or days for some trajectories



JIMO, Phase 2 of 3: Callisto to Ganymede



“The Jupiter Icy Moons Orbiter Reference Trajectory”, Gregory J. Whiffen, Try Lam, AAS 06-186



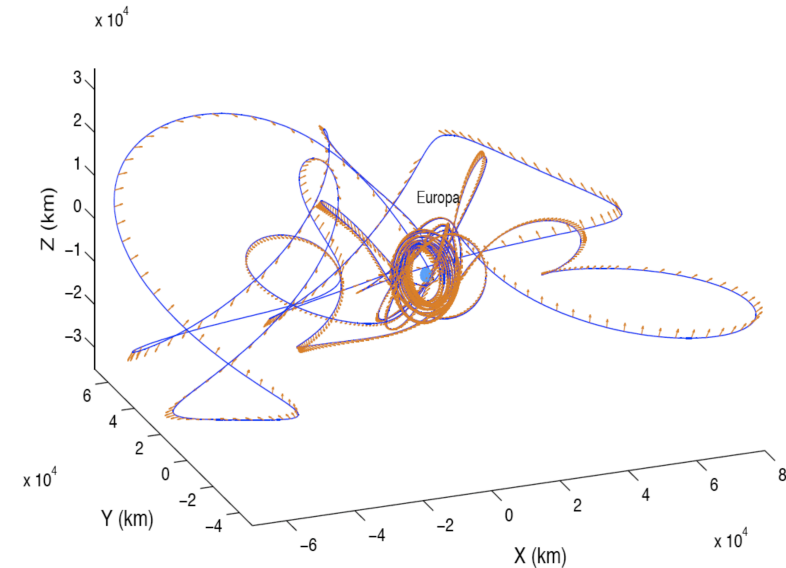
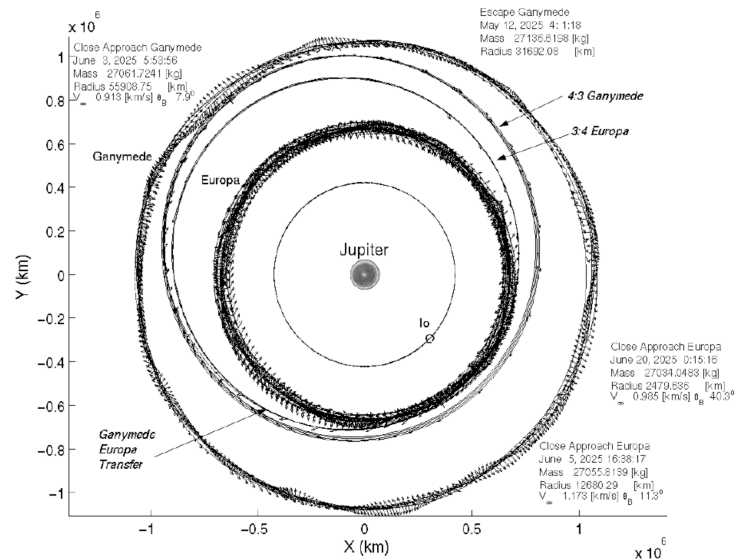
- Spiral up to high altitude DRO around Callisto which is “adjacent to” significantly larger and smaller 2-body like orbits around Jupiter when compared to Callisto’s orbit, allowing very little thrust to leave Callisto and enter much larger or smaller orbit around Jupiter than Callisto’s (see Whiffen and Lam for a discussion).
- We will revisit this in the context of dynamical systems theory for the 3 body problem - *optimal solutions computed by Mystic have been discovered to follow invariant manifolds of the dynamics!!*
- Suggests a method for initial trade studies of a large collection of trajectories - patch together trajectory segments living on the invariant manifolds which match target boundary conditions! We will return to this idea...



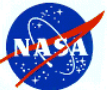
JIMO, Phase 3 of 3: Ganymede to Europa



“The Jupiter Icy Moons Orbiter Reference Trajectory”, Gregory J. Whiffen, Try Lam, AAS 06-186



- Goal for the Ganymede to Europa transfer different - need shortest flight time possible to minimize radiation hazard
- If assume thrusters on spacecraft, now is the time to use them!
- There do appear to be short (enough) flight times without the use of thrusting, but stability of these trajectories a major concern, and could quickly degenerate outside the control authority of the ion engine alone.
- **Note the engineering trade-off's encountered for this phase of mission design - the entire "Pareto front" of fuel and time optimal trajectories useful in trajectory design which achieve science goals with acceptable risk**



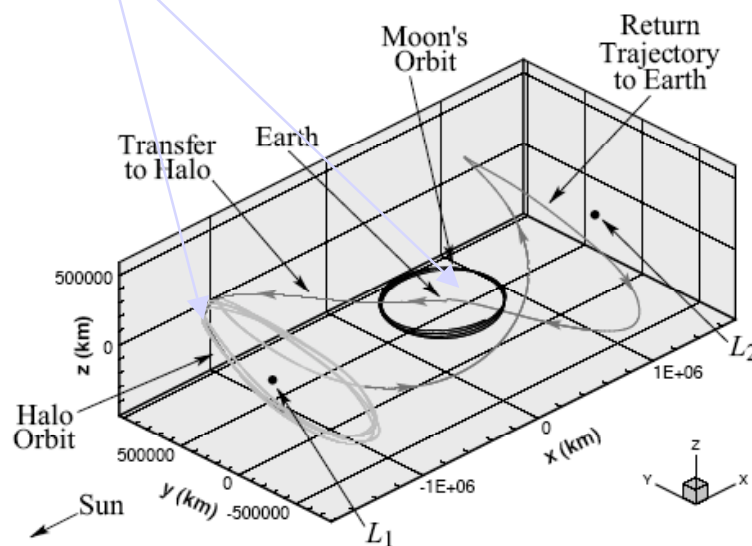
Overview of Optimal Control

$$J = \underbrace{\left(\sum_i v_i \cdot x(t_i) \right)}_{\text{BC's and "Science Waypoints"}} + \underbrace{\sum_j \int_{t_j}^{t_{j+1}} dt}_{\text{Cost of control required}} \underbrace{\left[L(x, u, t) + \lambda(t) \cdot (f(x, u, t) - \dot{x}) \right]}_{\text{Dynamical Constraints}}$$

BC's and "Science Waypoints"

Cost of control required

Dynamical Constraints



Solution! Compute:

$$0 = \dot{x} - f(x, u, t)$$

$$0 = \dot{\lambda} + \left(\frac{\partial f}{\partial x} \right) \lambda + \frac{\partial L}{\partial x}$$

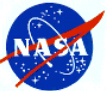
$$0 = \frac{\partial L}{\partial u} + \lambda \frac{\partial f}{\partial u}$$

A two-point BVP: Computational Soln. NOT trivial for nonlinear systems!!

Computational Challenges in Trajectory Design



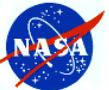
- Many local “minima” of objective
- Many solutions “almost equally good” but with different flight times, stability, or other properties potentially impacting mission design (example - JIMO design study)
 - Different properties of trajectories of nearly same cost is crucial for various phases of the JIMO mission concept. Some trajectories are much more unstable than others, leading to an increased risk of impact on the target moon if loss of ion engine. Other phases, for example the Ganymede to Europa transfer, required a shorter transit time at the expense of higher fuel cost to minimize exposure to high radiation levels.
 - Flexibility in mission design provided by the “Pareto front” of fuel-time near optimal trajectories.
- Motivates (or even requires) initial trade study and survey of wide class of trajectories in early phases of mission design.



Approach Implemented in EPISODE

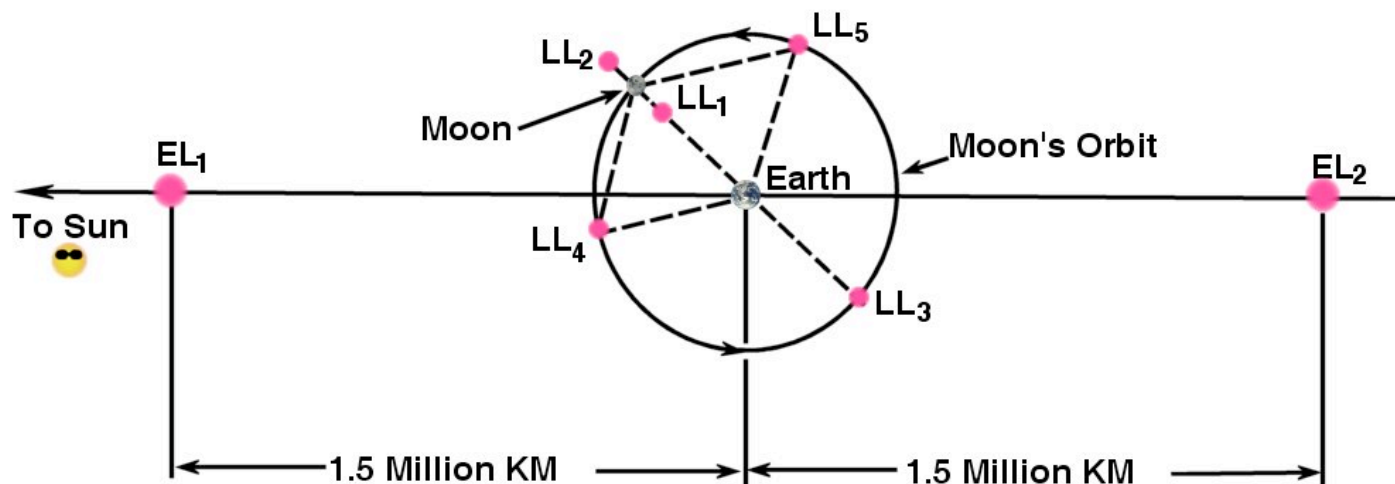


- Our Goal - a tool for broad initial trade studies, where a large class of possible trajectories can be generated and examined for possible advantages, and done in a semi-automatic fashion, *provably converging onto sets of progressively smaller objective function (fuel or time)*.
- Our ultimate infusion goal - to demonstrate capacity to efficiently generate many possible trajectory scenarios as input to high-fidelity optimization, with the goal of maximizing the number of trajectories as function of accuracy and computational expense.
- Strategy - we will construct a probability on trajectories, and use various algorithmic approaches to sample from it.
- Elements of this approach - Quickly generated trial trajectories (in general VERY far from optimal but possibly the right qualitative character), generation of “interesting waypoints”, use of trial waypoints for low-fidelity trade studies implemented in an MCMC framework, annealing, and finally hand off to high-fidelity global optimization (over the mission duration, not over space of all paths!)
- Specific strategy - LT by definition means that the optimal trajectory “shadows” segments of exact solutions. Suggests a fast method of generating an initial set of (very far from optimal) trajectories - integrate forward and backward, patch together (how?), take the “cloud of intersecting points” as a set of intermediate waypoints, and continue hierarchically.
- Moreover - the optimal solutions computed by Mystic have been empirically determined to follow along invariant manifolds! We will exploit insight into phase space structure gained from dynamical systems theory to quickly generate trial trajectories for phases of mission design well approximated as 3-body (or patched 3-body) dynamics.
- Question - the ultimate in low thrust are the “free trajectories” - what is the reachable set of the network of the underlying phase space flow?? Dynamical systems theory provides insight into this global structure...



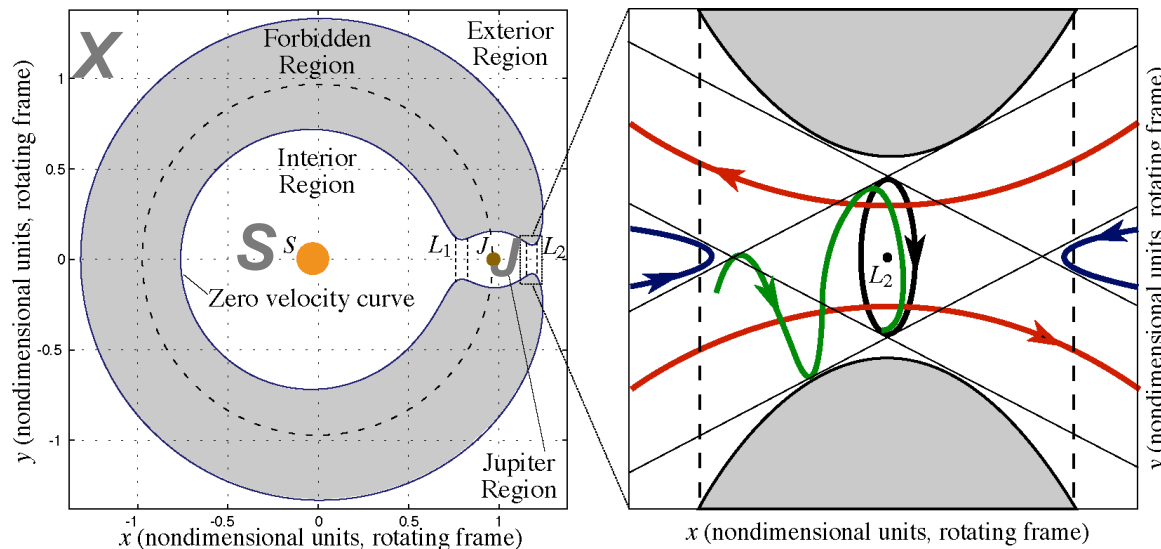
Fixed Points, and Associated Structures

- Every 3 Body System Has 5 Fixed Points Called Lagrange Points
 - Earth-Moon-S/C: LL_1, LL_2, \dots, LL_5
 - Sun-Earth-S/C: EL_1, EL_2, \dots
- They Generate the InterPlanetary Superhighway through:
 - Fixed points
 - Lyapunov orbits (in planar restricted problem)
 - Unstable and stable manifolds, and connections





Orbital Zoology Near the Lagrange Points



S: Sun Region

J: Jupiter Region

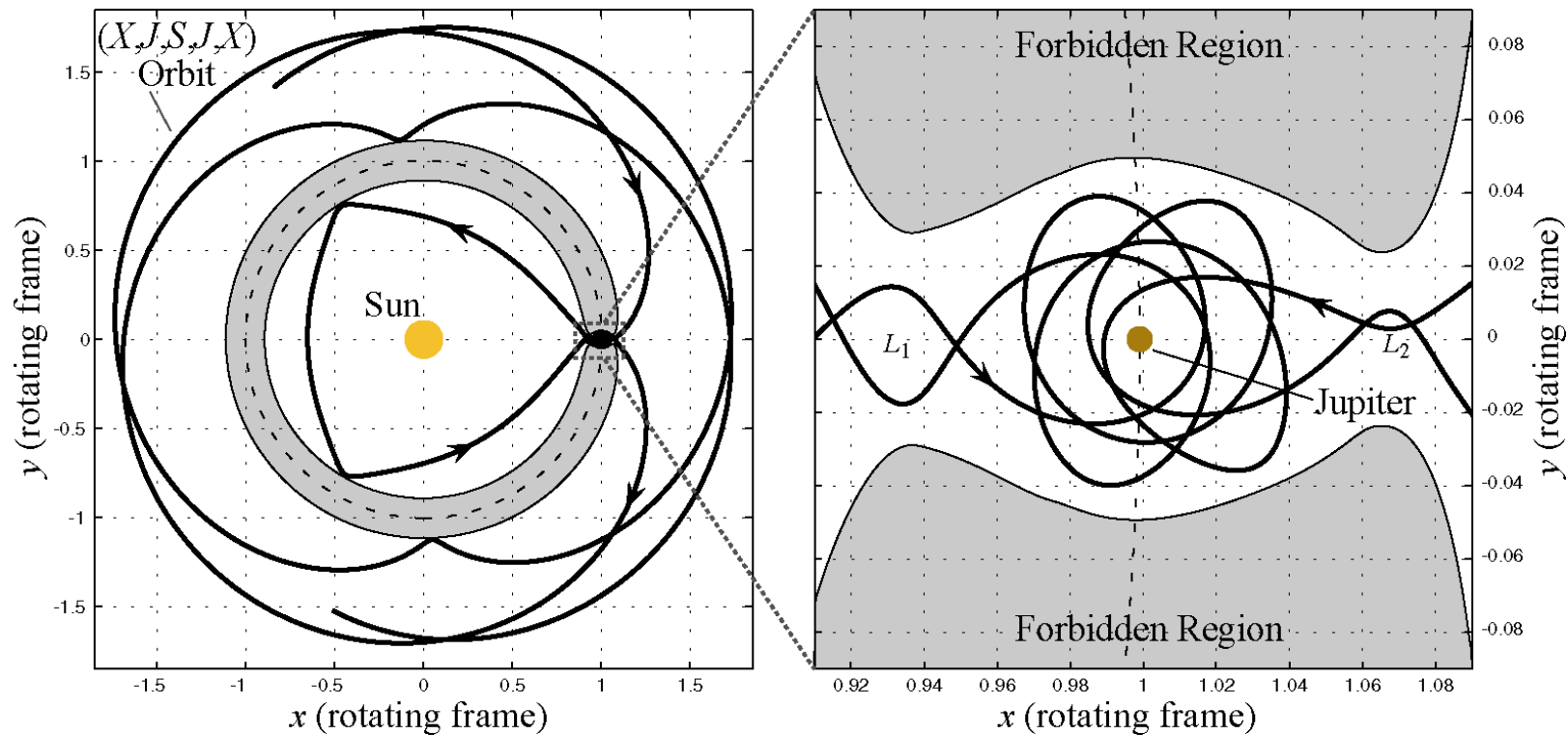
X: Exterior Region

(Outside Jupiter's Orbit)

Four Families of Orbits:

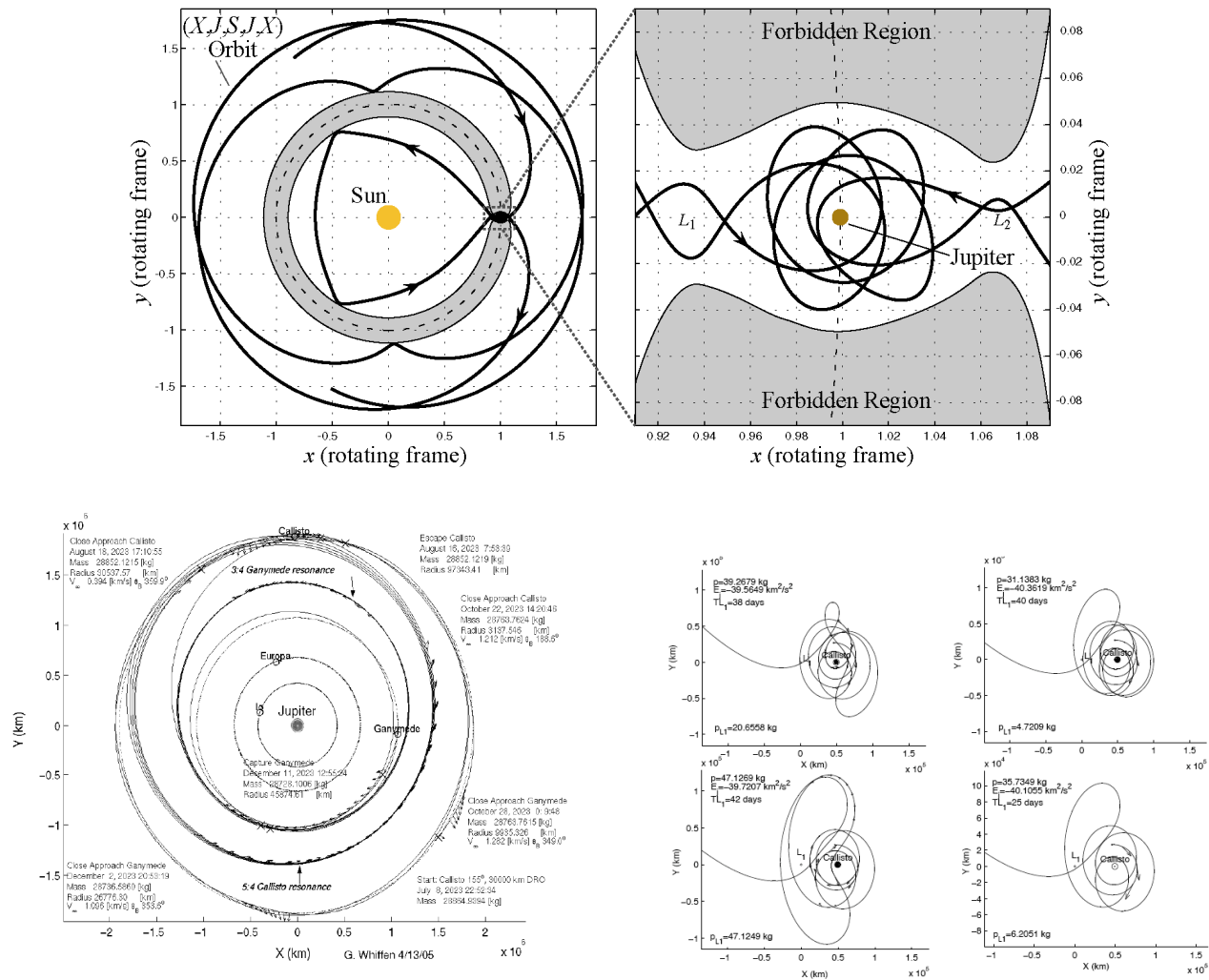
- Periodic Orbit (Planar Lyapunov)
- **Spiral Asymptotic Orbit** (Stable Manifold Pictured)
- Transit Orbits (MUST PASS THRU LYAPUNOV ORBIT)
- Non-Transit Orbits (May Transit After Several Revolutions)

Orbit with Itinerary $(X,J;S,J,X)$



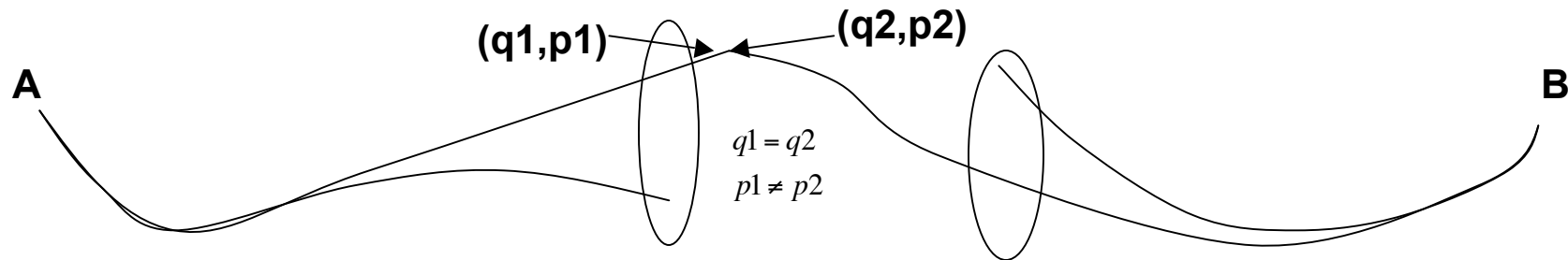
- Using Symbolic Dynamics Technique to Realize Complex Itinerary
- Capture Around Jupiter Multiple Revolutions (Specifiable)
- Note (2:3) to (3:2) Resonance Transition

Invariant Manifolds Provide Optimal Transfers!



“Model” of Low-Thrust Trajectories

$$\|X_{LT}(t) - X_I(t)\| \leq Ce^{Kt} \|\dot{X}_{LT} - F(X_{LT})\|$$



Integrate forward from A: Since thrust is bounded (and small), have an upper bound distance from (q1,p1) of free trajectory!

Low-thrust trajectory MUST at some point converge onto “free” trajectory as approach A backward in time.

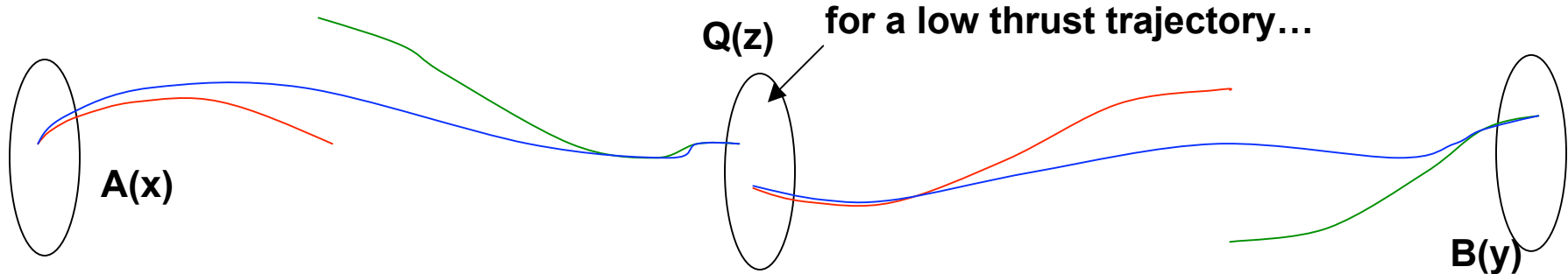
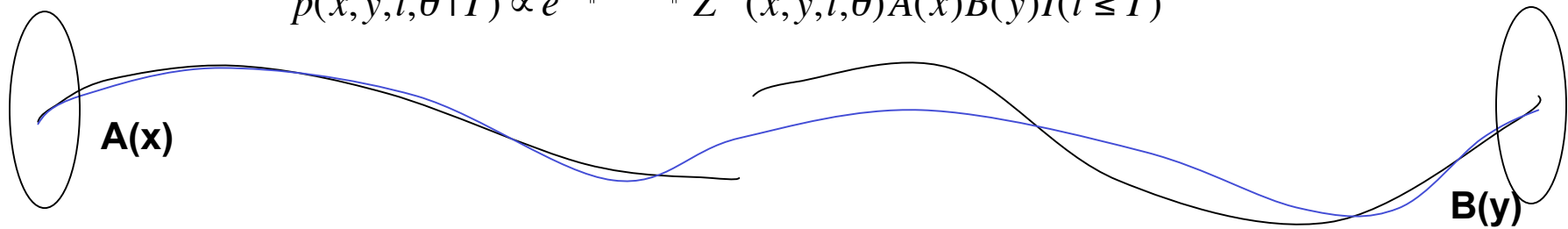
Integrate backward from B: Since thrust is bounded (and small), have an upper bound distance from (q2,p2) of free trajectory!

Low-thrust trajectory MUST at some point converge onto “free” trajectory as approach B forward in time.

Low-thrust trajectories must shadow the underlying flow of Hamiltonian dynamics for time intervals, then “restart” and shadow a new free trajectory!!

Probabilistic Exploration...

$$p(x, y, t, \theta | T) \propto e^{-\theta \|\dot{x} - F(x)\|^2} Z^{-1}(x, y, t, \theta) A(x) B(y) I(t \leq T)$$



$$p(x, y, t, \theta | T) \propto e^{-\theta \|\dot{x} - F(x)\|^2} Z^{-1}(x, y, t, \theta) A(x) Q(y) I(t \leq T)$$

$$p(x, y, t, \theta | T) \propto e^{-\theta \|\dot{x} - F(x)\|^2} Z^{-1}(x, y, t, \theta) Q(x) B(y) I(t \leq T)$$

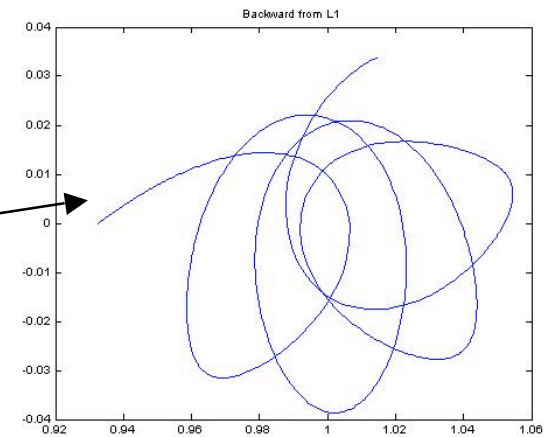
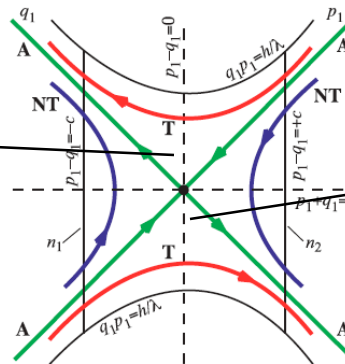
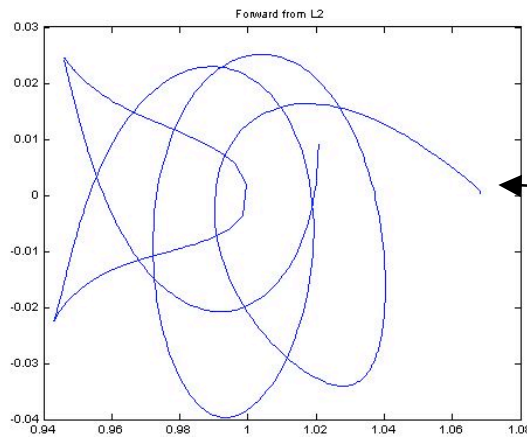
Patching Manifold Segments

$$J = \underbrace{\left(\sum_i \nu_i \cdot x(t_i) \right)}_{\text{BC's and "Science Waypoints":}} + \underbrace{\sum_j \int_{t_j}^{t_{j+1}} dt \left[\|u(t)\|^2 \right]}_{\text{Quadratic Cost}} + \underbrace{\lambda(t) \cdot ([J(\hat{x})](x - \hat{x}) - \dot{x})}_{\text{Linearized Dynamics}}$$

BC's and "Science Waypoints":
Here, L2 to L1 transfer...

Quadratic Cost

Linearized Dynamics



Solution: Find $u(t)$ so $x(t)$ satisfies BC's. If no Jump discontinuity, $u(t)=0$ and follow reference segments

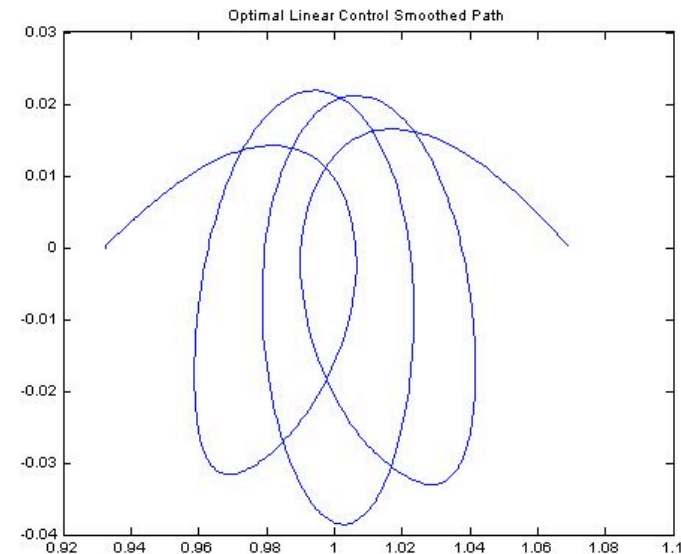
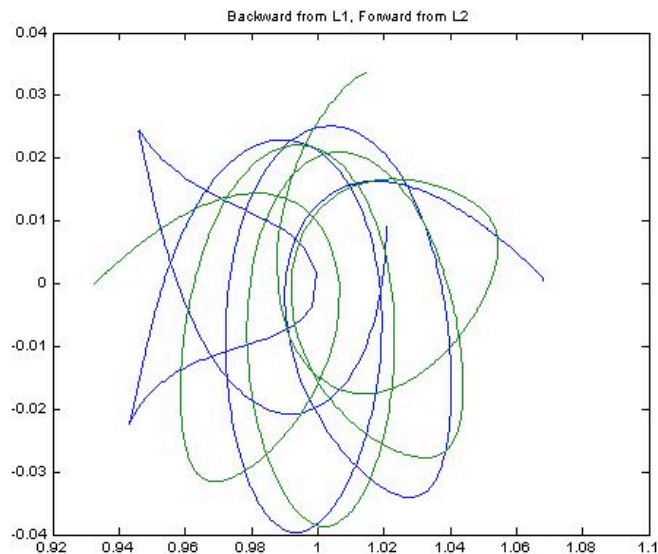
Optimal Linear Control

$$J = \underbrace{\left(\sum_i v_i \cdot x(t_i) \right)}_{\text{BC's and "Science Waypoints":}} + \sum_j \int_{t_j}^{t_{j+1}} dt \underbrace{\left[\|u(t)\|^2 \right]}_{\text{Quadratic Cost}} + \underbrace{\lambda(t) \cdot ([J(\hat{x})](x - \hat{x}) - \dot{x})}_{\text{Linearized Dynamics}}$$

BC's and "Science Waypoints":
Here, L2 to L1 transfer...

Quadratic Cost

Linearized Dynamics



OLC path now can be used as an initial guess for subsequent optimization (MCMC, followed by deterministic methods)

Multiple Shooting Method

Multiple shooting solves a two-point boundary value problem by simultaneously solving initial value problems at several intermediate points and piecing the functions together:

Problem : find solution $y(x)$ that satisfies

$$y' = f(x, y) \text{ and } f(a) = A, f(b) = B$$

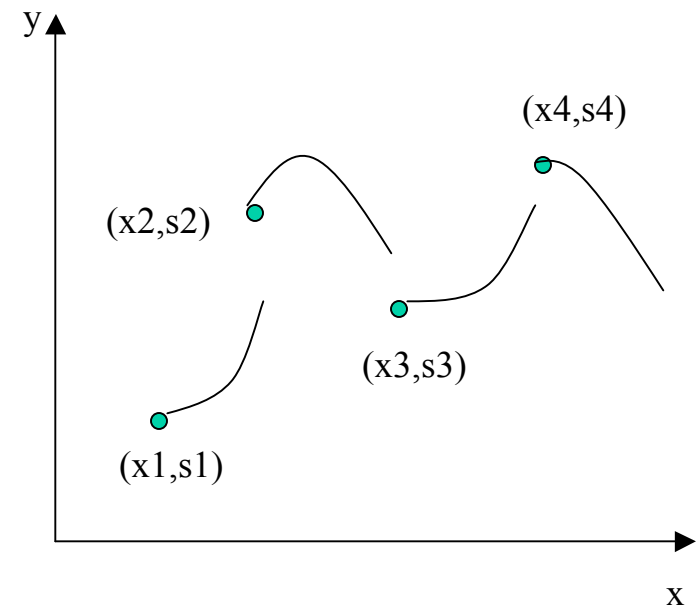
Method : determine the vector s_k , $k = 1, 2, \dots, m$ that makes the function pieced together by $y(x; x_k, s_k)$ continuous

$$y(x) := y(x; x_k, s_k) \text{ for } x \in [x_k, x_{k+1})$$

$y(x; x_k, s_k)$ is the solution of the initial-value problem of

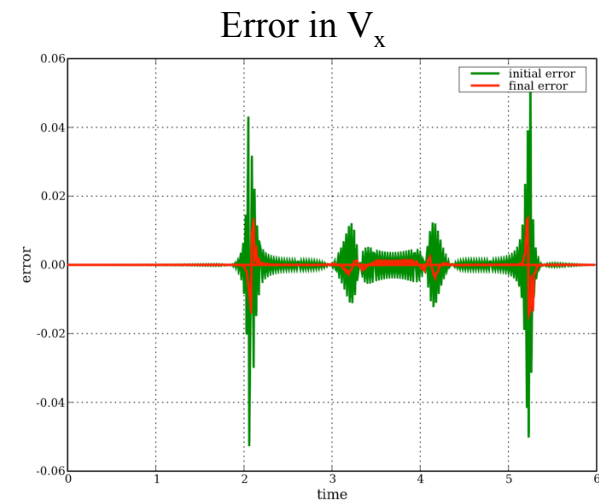
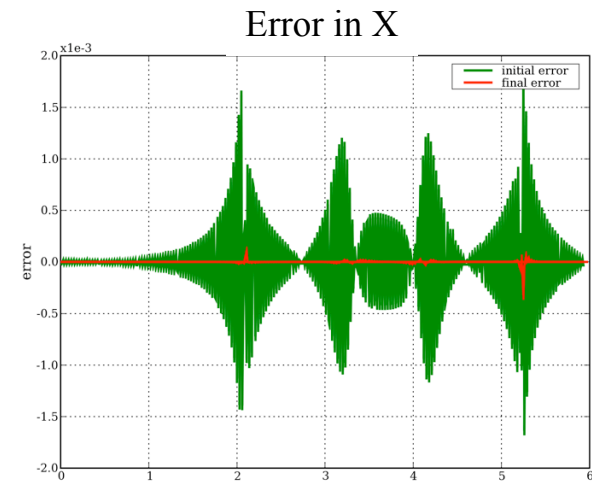
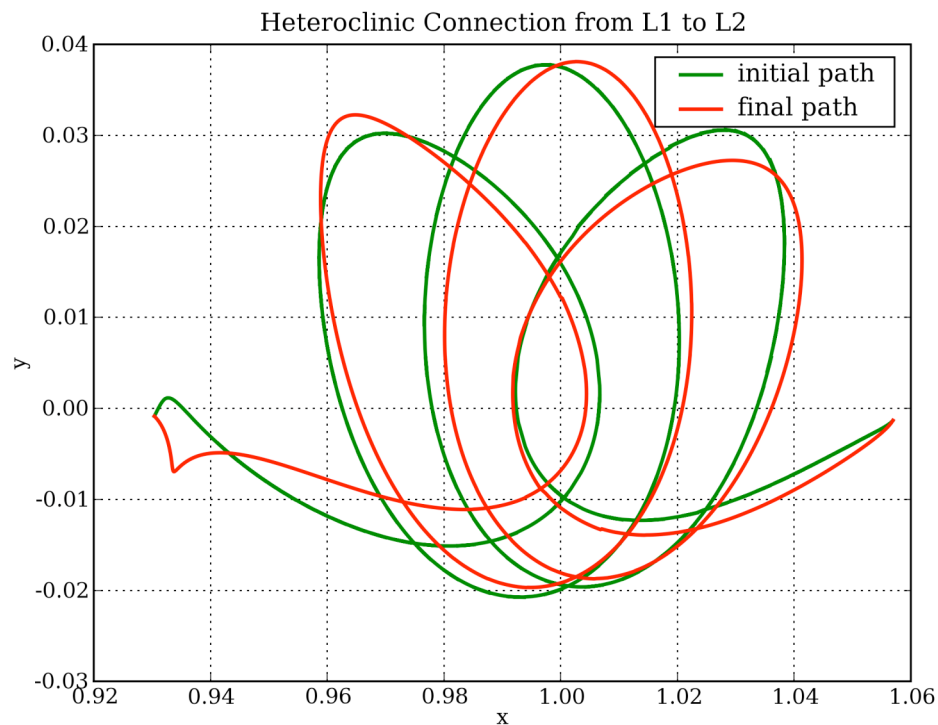
$$y' = f(x, y), y(x_k) = s_k$$

$$a = x_1 < x_2 < x_3 < \dots < x_m = b$$



Multiple Shooting Result

The smoothed path generated by the smoothing code is used as an initial path for the multiple shooting.
Multiple shooting reduces the position error by factor of 10 from 0.002 to 0.0002.
Multiple shooting reduces the velocity error by factor of 2 from 0.05 to 0.028.



Modular Design of EPISODE



User supplied routine: $\dot{x} = F(x)$

User supplied:

Regions of interest and routines to generate BC's "in the vicinity"

i.e. catalog of periodic Science orbits around Planets, moons, L pts, etc.

```
Program sample_mission_soln
  If (not(init_waypoints)) then
    generate_waypoints
  End if
  Call sample_initial_ensemble()
  Call calibrate_initial_ensemble()
  Do I=1,num_hierarchies
    Call calibrate_ensemble(level,paths)
    Call mcmc_paths(level,niter,paths)
    If (I < num_hierarchies) then
      Call fast_deterministic_step()
    End if
  End do
  Call high_fidelity_optimizer()
End program sample_mission_soln
```

EPISODE (core set of routines):

Dynamics interface/

- Evaluates F, Jac, etc.

Error_functional/

- Evaluates log p for MCMC

Inference_tools/

- fit parameters for MCMC proposals

lo_tools/

Linearized_dynamics/

- Fixed point analysis

Mcmc/

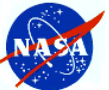
- drivers, and proposals based on
variational approach to trajectory generation

Variational_problems/

Utils/

Libraries:

Adolc/ Cfitsio/ DLSODE/ LAPACK/



Summary



- A new era of mission design is here, utilizing 2 recent advances: 1) IPS, and 2) low thrust. These are enabling technologies for entirely new mission concepts such as JIMO. *Now time to perfect the art of optimal open and closed loop control utilizing these mission enabling technologies.*
- Design of low-thrust trajectories challenging - Many solutions “almost equally good” but with different flight times, stability, or other properties potentially impacting mission design (example - JIMO design study)
 - Different properties of trajectories of nearly same cost can be crucial in achieving mission objectives while mitigating risk (I.e. impact if loss of control)
 - Other phases, for example the Ganymede to Europa transfer, required a shorter transit time at the expense of higher fuel cost to minimize exposure to high radiation levels.
 - Flexibility in mission design provided by the “Pareto front” of fuel-time near optimal trajectories.
 - Exploration of entire ensemble of (low-fidelity) trajectories very important in early phases of mission design
- Our Goal - use a probabilistic framework (implemented computationally with the code EPISODE) for initial trajectory design studies, where a large class of possible trajectories can be generated in a semi-automatic fashion, *provably converging onto sets of progressively smaller objective function (fuel or time).*
- Strategy - guide trajectory generation, when possible, with insight provided by dynamical systems theory
 - Quickly generated trial trajectories (in general VERY far from optimal but possibly the right qualitative character)
 - generation of “interesting waypoints” for low-thrust transfers
 - use of trial waypoints to quickly sample “second generation” of trajectories for low-fidelity trade studies (implemented with MCMC)
 - “Annealing” (take small number of deterministic steps from MCMC output to initialize a new ensemble).
 - Hand off to high-fidelity global optimization (over the mission duration, not over space of all paths!)
- Status and future work
 - Core collection of routines completed
 - Initial numerical work for solving variational problem for fast trajectory generation and improvement completed
 - MCMC driver routines written, and initial experiments with “Gibbs sampling” type proposals (FAILED to have sufficient mixing, and experimenting with other strategies now, with goal a working approach by end of FY08)
 - Expected completion of 3 body trajectory generation targeted for end of FY08 (including “Gensis” design application)

